Implementation of an Edge-Based H-Formulation in the Nonlinear Magnetostatic Case

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Keywords: Finite element method, $\boldsymbol{H}$-formulation, magnetostatics, nonlinear magnetic field analysis

Abstract

This paper deals with the usage of a magnetic field oriented formulation that is derived from a mixed formulation using the penalty method, which reduces the problem to one partial differential equation. To achieve accurate results the occurring penalty factor has to be chosen properly. For this reason, a rule to select this parameter is developed for linear and nonlinear material laws. It is shown that it yields good results when compared to measurements provided by the TEAM problem 13.

1 Introduction

Formulations based on the magnetic field intensity have been a big topic in research [1], since they use the magnetic field intensity as a main unknown and therefore give direct access to the field quantities. One particular formulation of this kind is a mixed formulation [2]. The major drawback of this formulation is its large number of unknowns, since both involved quantities are vectors. For this reason, a penalty method is applied to eliminate one unknown and end up in one equation that only keeps the degrees of freedom of the magnetic field intensity [3]. The used perturbation technique needs a penalty factor that has to be chosen appropriately to obtain correct results. In this paper, a rule to choose this parameter is developed in the linear and nonlinear case. Furthermore, the technique how to solve the resulting nonlinear equations is shown using the Newton-Raphson method combined with an exact line search.

2 Finite Element Formulation

In this paper, a computational domain Ω is considered where a magnetostatic field problem is solved. The boundary of the region is described by $\partial\Omega = \Gamma_1 \cup \Gamma_2$ with an magnetic flux density boundary condition $\boldsymbol{B}\cdot\boldsymbol{n} = 0$ on Γ_1 and a magnetic field intensity boundary condition $\boldsymbol{n}\times\boldsymbol{H} = 0$ on Γ_2.

2.1 Mixed Formulation

Introducing a magnetic vector potential $\boldsymbol{B} = \nabla\times\boldsymbol{A}$ to fulfil $\nabla\cdot\boldsymbol{B} = 0$ and using Ampéres law $\nabla\times\boldsymbol{H} = \boldsymbol{J}$, where $\boldsymbol{J}$ is the source current density, yields a mixed formulation for the magnetostatic case: Find $(\boldsymbol{H},\boldsymbol{A}) \in H(\mathrm{curl},\Omega) \times H_0(\mathrm{div}^0,\Omega)$ such that

$$\int_\Omega \boldsymbol{B}(\boldsymbol{H})\cdot\boldsymbol{H}' - \boldsymbol{A}\cdot\nabla\times\boldsymbol{H}' \,\mathrm{d}V = 0 \ , \tag{1a}$$

$$\int_\Omega (\nabla\times\boldsymbol{H} - \boldsymbol{J})\cdot\boldsymbol{A}' \,\mathrm{d}V = 0 \ , \tag{1b}$$

$$\forall\boldsymbol{H}' \in H(\mathrm{curl},\Omega) \text{ and } \forall\boldsymbol{A}' \in H_0(\mathrm{div}^0,\Omega) \ .$$

Imposing a gauge condition using Lagrange multipliers results in a third equation which increases the number of unknowns.

2.2 Penalty Formulation

Adding the term $-(1/\rho)\int_\Omega \boldsymbol{A}\cdot\boldsymbol{A}' \,\mathrm{d}\Omega$ to (1b) gives

$$\boldsymbol{A} = \rho(\nabla\times\boldsymbol{H} - \boldsymbol{J}) \ , \tag{2}$$

which can be substituted into (1a) to obtain the penalized version of the mixed formulation: Find $\boldsymbol{H} \in H(\mathrm{curl},\Omega)$ such that

$$\int_\Omega \boldsymbol{B}(\boldsymbol{H})\cdot\boldsymbol{H}' + \rho\nabla\times\boldsymbol{H}\cdot\nabla\times\boldsymbol{H}' \,\mathrm{d}V = \int_\Omega \rho\boldsymbol{J}\cdot\nabla\times\boldsymbol{H}' \,\mathrm{d}V \quad \forall\boldsymbol{H}' \in H(\mathrm{curl},\Omega) \ . \tag{3}$$

In the nonlinear case, the Newton-Raphson method is used. In doing so, the following system of equations has to be solved

$$\frac{\partial\mathscr{R}_i}{\partial\boldsymbol{H}_i}\Delta\boldsymbol{H} = -\mathscr{R}_i \ , \quad \boldsymbol{H}_{i+1} = \boldsymbol{H}_i + \eta\Delta\boldsymbol{H} \ . \tag{4}$$

In every Newton iteration i, the solution $\Delta\boldsymbol{H}$ is added to the previous one until the norm of the residual is small enough ($\|\mathscr{R}\| < \epsilon = 10^{-12}$). In (4) $\mathscr{R}_i$ is the residual, $\partial\mathscr{R}_i/\partial\boldsymbol{H}_i$ is the Jacobian matrix of the underlying equation in the current Newton iteration and η is the line search parameter. The Jacobian is approximated by using the concept of the Fréchet-derivative and finite differences

$$\frac{\partial\mathscr{R}}{\partial\boldsymbol{H}}\Delta\boldsymbol{H} = \int_\Omega \boldsymbol{\mu}_\mathrm{d}\Delta\boldsymbol{H}\cdot\boldsymbol{H}' + \rho\nabla\times\Delta\boldsymbol{H}\cdot\nabla\times\boldsymbol{H}' \,\mathrm{d}V \ ,$$

where the resulting matrix $\boldsymbol{\mu}_\mathrm{d} = \partial\boldsymbol{B}(\boldsymbol{H})/\partial\boldsymbol{H}$ is obtained from the given material law.

3 Implementation

With regard to the proposed formulation, two aspects of the implementation need to be highlighted. These include an appropriate choice of the penalty factor used in the formulation and a line search technique that improves convergence.

3.1 Choice of the Penalty Factor

The aim of this research is to provide a rule of thumb for direct solvers, where the accuracy of the solution remains the same over a large range of the system matrix condition number. To do so, the following formula as a rule of thumb (r.o.t.) is proposed in the linear case

$$\rho = \frac{2}{p^{0.01}} \mu_{\mathrm{r,max}} \mu \cdot 10^{p/2} \,, \tag{5}$$

where p is chosen small ($p = 10^{-6}$). Figure 1 shows that, when no rule is applied and simulations are performed over a broad range of values ($10^{-12} \le \rho \le 10^{8}$) and compared with the mixed formulation, a better accuracy can be achieved, but it is difficult to choose exactly this value. The scaling with the permeability μ and $\mu_{\mathrm{r,max}}$ allows choosing the factor such that the accuracy remains constant.

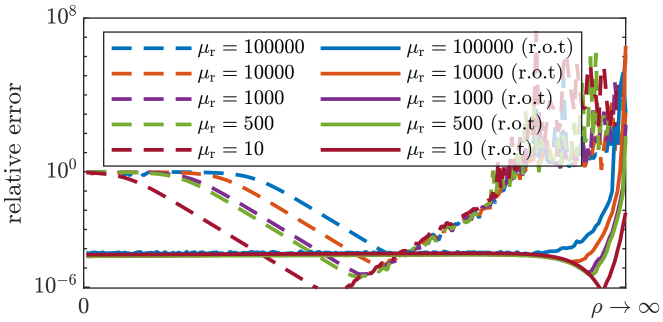

Fig. 1: Relative error resulting from the comparison of the mixed formulation without (dashed line) and with the proposed rule (full line) over a range of different linear permeabilities.

For nonlinear materials, this rule remains the same, except that $\mu_{\mathrm{r,max}}$ is set to 1, since the maximum value of the relative permeability depends on the state of the system. To obtain useful values for the permeability, the following relation is evaluated

$$\mu = \mathrm{norm}\left(\frac{\partial \boldsymbol{B}(\boldsymbol{H})}{\partial \boldsymbol{H}}\right) \,, \tag{6}$$

where $\mathrm{norm}(\cdot)$ corresponds to the largest singular value of the matrix.

3.2 Determination of the Line Search Parameter

The Newton-Raphson method only converges if an appropriate starting value is chosen. Since this is most of the time unknown, line search methods are used to ensure convergence to the solution independent of the starting point. In this work, an exact line search is used where in every Newton iteration the energy functional $\mathscr{E}$, which minimizer is the solution of the problem, is minimized along the Newton direction $\Delta \boldsymbol{H}$. The energy functional $\mathscr{E}$ is derived w.r.t. η which gives a first order optimality condition

$$\frac{\partial \mathscr{E}_{i+1}}{\partial \eta_i} = \left(\frac{\partial \mathscr{E}_{i+1}}{\partial \boldsymbol{H}_{i+1}}\right) \frac{\partial \boldsymbol{H}_{i+1}}{\partial \eta_i} = \mathscr{R}(\eta)^{\top} \Delta \boldsymbol{H}_i \overset{!}{=} 0 \,. \tag{7}$$

The root of the resulting equation, which just consists of the residual multiplied with the current Newton direction, is solved by using Brent's method [4].

4 Results

To verify the implementation of the formulation, it is tested on the TEAM problem 13 [5], where a racetrack shaped coil is turned around some steel structure, with negligible eddy current effect. In the implementation, the provided BH-curve is approximated using B-splines, which offers techniques to obtain $\partial \boldsymbol{B}(\boldsymbol{H})/\partial \boldsymbol{H}$ for the Newton-Raphson method and for the penalty factor. In Figure 2 it can be seen that the simulation results match the measurements quite good.

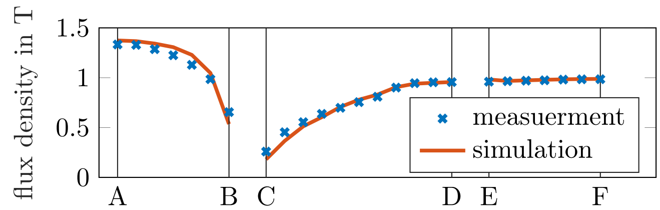

Fig. 2: Comparison between measurement and simulation for TEAM problem 13 in different parts of the steel sheet.

5 Conclusion

In this paper, the penalized version of the mixed formulation is solved in the nonlinear case. It is shown that there is a working rule of thumb in the linear and nonlinear case for selecting a proper value for the penalty factor that gives comparable results when compared to the initial formulation. Furthermore, a line search method that minimizes an energy functional along the Newton direction is employed, to obtain an improved convergence behavior. In the full paper, this concept will be extended to the eddy current problem and hysteretic case, where this formulation seems to be a good candidate to incorporate hysteresis models that use the magnetic field intensity as the input.

Acknowledgements

This work was supported by the joint DFG/FWF Collaborative Research Centre CREATOR (DFG: Project-ID 492661287/TRR 361; FWF: 10.55776/F90).

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